13 B.

1. a.) i. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $t \in \mathbb{R}$

ii. $x = 3 + t$, $y = -4 + 4t$, $t \in \mathbb{R}$

iii. $t = \frac{x-3}{1} = \frac{y+4}{4} \Rightarrow 4x - 12 = y + 4$

$\boxed{4x - y = 16}$

b.) i. If the line has direction vector b ~~perp~~ perpendicular to $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$, then

$b \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 0$ so, $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ seems reasonable.

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 5 \end{pmatrix}$, $t \in \mathbb{R}$

ii. $x = 5 - 2t$, $y = 2 + 5t$, $t \in \mathbb{R}$

iii. $t = \frac{x-5}{-2} = \frac{y-2}{5} \Rightarrow 5x - 25 = -2y + 4$

$\boxed{5x + 2y = 29}$

c.) i. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ $t \in \mathbb{R}$

ii. $x = -6 + 3t$, $y = 7t$, $t \in \mathbb{R}$

iii. $t = \frac{x+6}{3} = \frac{y}{7} \Rightarrow 7x + 42 = 3y$

$\boxed{7x - 3y = -42}$

d.) i.) Take $(-1, 11)$ as our fixed point. So, $a = \begin{pmatrix} -1 \\ 11 \end{pmatrix}$
the direction vector $b = \begin{pmatrix} -3-(-1) \\ 12 - 11 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

ii.) $x = -1 - 2t$, $y = 11 - t$, $t \in \mathbb{R}$

iii) $t = \frac{x+1}{-2} = y - 11 \Rightarrow x + 1 = -2y + 22$

$\boxed{x + 2y = 21}$

2. a) $x = -1 + 2t$, $y = 4 - t$. $t \in \mathbb{R}$

b) When

t	Process for x	Process for y	Point
0	$x = -1 + 2(0) = -1$	$y = 4 - 0 = 4$	$(-1, 4)$
1	$x = -1 + 2(1) = 1$	$y = 4 - 1 = 3$	$(1, 3)$
3	$x = -1 + 2(3) = 5$	$y = 4 - 3 = 1$	$(5, 1)$
-1	$x = -1 + 2(-1) = -3$	$y = 4 - (-1) = 5$	$(-3, 5)$
-4	$x = -1 + 2(-4) = -9$	$y = 4 - (-4) = 8$	$(-9, 8)$

3. a) $t + 2 = 3$ and $1 - 3t = -2$

then $t = 1$ and $-3t = -3$

$t = 1$

Since $t = 1$ in each case, $(3, -2)$ lies on the line.

b.) If $(k, 4)$ lies on $x = 1 - 2t$, $y = 1 + t$

then

$k = 1 - 2t$, $4 = 1 + t$

$t = 3$

and

$k = 1 - 6$

$k = -5$

4. a.) When $t = 1$, $r = \binom{1}{5} + 1 \times \binom{-1}{3}$

$= \binom{1-1}{5+3} = \binom{0}{8}$ the point is $(0, 8)$

b.) $\binom{1}{-3}$ is a non-zero scalar multiple of $\binom{-1}{3}$.

So, it could also be used to describe the direction of the line.

c.) the line passes through point $(0, 8)$ and has direction vector $\binom{1}{-3}$

$r = \binom{0}{8} + s\binom{1}{-3}$ is AN alternative vector equation for the line.

5. a) i. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} + \tau \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \tau \in \mathbb{R}$

ii. $x = 1 + 2t, \quad y = 3 + \tau, \quad z = -7 + 3\tau, \tau \in \mathbb{R}$

b) i. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad t \in \mathbb{R}$

ii. $x = t \quad y = 1 + \tau \quad z = 2 - 2t, \tau \in \mathbb{R}$

c.) i. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}$

ii. $x = -2 + t, \quad y = 2, \quad z = 1, \quad t \in \mathbb{R}$

d.) i. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad t \in \mathbb{R}$

ii. $x = 2t, \quad y = 2 - t, \quad z = -1 + 3t, \quad t \in \mathbb{R}$

6. a) $\overrightarrow{AB} = \begin{pmatrix} -1-1 \\ 3-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, t \in \mathbb{R}$

b.) $\overrightarrow{CD} = \begin{pmatrix} 3-0 \\ 1-1 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, t \in \mathbb{R}$

c.) $\overrightarrow{EF} = \begin{pmatrix} 1-1 \\ -1-2 \\ 5-5 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}, t \in \mathbb{R}$

d.) $\overrightarrow{GH} = \begin{pmatrix} 5-0 \\ -1-1 \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}, t \in \mathbb{R}$

☆ IB Question

7. Given $x = 1 - t$, $y = 3 + t$, $z = 3 - 2t$

mona presents

a) the line meets the XOY plane when $z = 0$

So, $3 - 2t = 0$

$\boxed{t = 3/2}$

then, $x = 1 - \frac{3}{2} = -\frac{1}{2}$ and $y = 3 + \frac{3}{2} = \frac{9}{2}$

So the point is $(-\frac{1}{2}, \frac{9}{2}, 0)$

b.) the line meets YOZ plane when $x = 0$

So, $1 - t = 0$, $t = 1$ } then, $y = 3 + 1 = 4$, $z = 3 - 2 = 1$ } point $(0, 4, 1)$

c.) The line meets the XOZ plane when $y = 0$

So, $3 + t = 0$, $t = -3$ } Then, $x = 1 - (-3) = 4$, $z = 3 - 2(-3) = 9$

Point $(4, 0, 9)$

8. Given a line with equations

$x = 2 - t$

$y = 3 + 2t$

$z = 1 + t$

the distance to the point $(1, 0, -2)$ is:

$\sqrt{(2-t-1)^2 + (3+2t-0)^2 + (1+t+2)^2}$

But the distance is $= 5\sqrt{3}$ units

so, $\sqrt{(1-t)^2 + (3+2t)^2 + (t+3)^2} = 5\sqrt{3}$ } square both sides

$(1-t)^2 + (3+2t)^2 + (t+3)^2 = 75$

$1 - 2t + t^2 + 9 + 12t + 4t^2 + t^2 + 6t + 9 = 75$

$6t^2 + 16t - 56 = 0$ } factor out a 2.

$3t + 8t - 28 = 0$

$(\cancel{3t+14}\ (3t+14)(t-2) = 0$

$t = -14/3$ or $t = 2$

when $t = 2$, the point is $(0, 7, 3)$

when $t = -14/3$, the point is

$(20/3, -19/3, -11/3)$